

Bose-Einstein condensation of magnons under incoherent pumping

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Bose-Einstein condensation in a gas of magnons pumped by an incoherent pumping source is experimentally studied at room temperature. We demonstrate that the condensation can be achieved in a gas of bosons under conditions of incoherent pumping. Moreover, we show the critical transition point is almost independent of the frequency spectrum of the pumping source and is solely determined by the density of magnons. The electromagnetic power radiated by the magnon condensate was found to scale quadratically with the pumping power, which is in accordance with the theory of Bose-Einstein condensation in magnon gases.

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Several decades after the phenomenon of Bose-Einstein condensation (BEC) has been predicted for a gas of atoms with an integer spin (bosons) [1], it was understood that bosonic elementary excitations in solids, quantum liquids and giant molecules can undergo the same transition [2, 3, 4]. BEC takes place, if the density of (quasi)-particles is larger than a critical value, N_c , which increases with increasing temperature of the system. In this connection the advantage of bosons in solids in comparison with real Bose atoms is that their density can be substantially increased by external pumping and therefore the BEC transition can be reached at higher temperatures. Experimental observation of BEC in ensembles of laser pumped excitons [5] and polaritons [6, 7] as well as in the gas of parametrically pumped magnons [8] were reported at temperatures, which are much higher than those for atomic gases. However, since bosonic quasi-particles in solids are pumped by an external source, the source might introduce an additional coherence in the system. This effect can mimic the spontaneous coherence intrinsic for BEC [9], in particular, if the pumping source is coherent, as it was the case in the above experiments.

In this Letter we investigate a gas of magnons pumped by an incoherent (noise) pumping source covering a relatively wide frequency interval. We report the formation of BEC in the ensemble of magnons pumped by such a source, which is a principal step showing that the coherent condensate of quasi-particles can be created by the magnons with randomly-distributed frequencies, wave-vectors and phases. We experimentally show that the critical point (which was documented by a vast increase (6-7 orders of magnitude) of the electromagnetic radiation from the condensate) is almost independent of the frequency spectrum of the pumping source and is solely determined by the density of magnons.

Possibility of BEC in a gas of magnons (the quanta of spin waves in magneto-ordered systems) under strong microwave pumping was considered about twenty years

ago [10, 11]. Recently BEC of quasi-equilibrium magnons was discovered in yttrium iron garnet (YIG) thin films at room temperature [8]. The magnons in this and several following experiments [12, 13, 14] were pumped by a coherent microwave through parametric excitation: a microwave photon of the frequency f_p creates a pair of primary magnons with the frequency, which is half of the pumping frequency and oppositely oriented wave vectors $f_{\mathbf{k}_p} = f_{-\mathbf{k}_p} = f_p/2$. If the microwave power exceeds a certain threshold, $P_1^0 \propto (\delta f_p)^2$, where δf_p is the magnon relaxation frequency, the number of parametric pairs grows dramatically in a narrow frequency interval around $f_p/2$. The primary magnons are “very hot”: their spectral density becomes several orders of magnitude larger than that of the thermal magnons at room temperature. Four-magnon scattering processes, which preserve the total number of magnons, redistribute the magnons over the magnon spectrum and create a quasi-equilibrium distribution of magnons outside of the resonance region.

The population function of magnons, as it follows from the theory [11] and is confirmed by the experiment [12], is the Bose-Einstein one with an effective temperature T and an effective chemical potential μ . Without pumping $\mu = 0$ and $T = T_{\text{lattice}}$ (room temperature in our case) due to thermal equilibrium between the magnon gas and the lattice. Applied pumping increases μ due to excitation of additional magnons. The value of μ is determined by the energy flow equilibrium between the microwave pumping and the spin-lattice relaxation of magnons. Thus, by changing the pumping power, one can control μ . On the other hand, the deviation of effective temperature from the lattice temperature can be neglected at room temperature as long as the total number of pumped magnons ($10^{18} - 10^{19} \text{ cm}^{-3}$ in our experiments) is much smaller than the total number of thermal magnons (10^{22} cm^{-3}). An incoherent pumping covering the frequency interval $f_p \pm \Delta f/2$ does not modify the physical picture of the parametric pumping dramati-

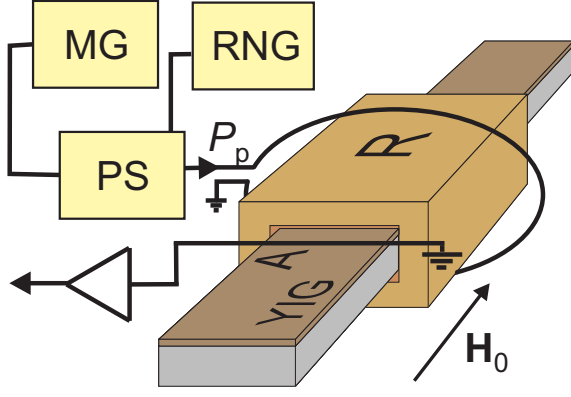


FIG. 1: (Color online) Schematic diagram of the used experimental setup. Microwave generator (MG) generates pulses of microwave radiation. Phase-shifter (PS) controlled by the random number generator (RNG) randomizes the phase of the radiation which is supplied to the dielectric resonator (R). The resonator applies the noise pumping power to the YIG film. Antenna (A) detects the electromagnetic radiation from the created condensate.

cally [11], merely increasing the pumping power threshold $P_1 \propto \delta f_p(\delta f_p + \Delta f)$ [15, 16, 17].

When μ reaches the bottom of the spin-wave spectrum $\mu_c = 2\pi\hbar f_{\min}$, where $f_{\min} = \min(f_{\mathbf{k}})$, the magnons at the lowest magnon state create a coherent Bose-Einstein condensate. It can be revealed either by enormous light scattering cross-section [13] or by direct detection of the electromagnetic radiation from the condensate [14].

The sketch of the experimental setup is shown in Fig. 1. The measurements were carried out on $20\text{ }\mu\text{m}$ thick epitaxial yttrium-iron-garnet (YIG) film with the lateral dimensions $1.5 \times 10\text{ mm}^2$. A uniform in-plane static magnetic field $H_0 = 0.8\text{ kOe}$ was applied along the long side of the sample. The corresponding $f_{\min} = 2.1\text{ GHz}$. Magnons were excited by the microwave field of an open dielectric resonator with resonance frequency $f_p = 9.4\text{ GHz}$. The microwave magnetic field was parallel to the static field to fulfill the conditions of the parallel pumping. The resonator was fed with microwave pulses with $40\text{ }\mu\text{s}$ duration and a repetition period of 20 ms. The incoherency of the pumping signal was realized by a random variation of its phase by $\pm\pi/2$ by means of a phase shifter, controlled by a random number generator. The phase was randomized with a frequency f_d , which determines the width of frequency spectrum of the pumping power $\Delta f \simeq 0.9f_d$. The width Δf was typically much greater than the relaxation frequency δf_p ($\sim 1 - 2\text{ MHz}$ in our experiment) determined by the magnon lifetime. The frequency-integrated peak pumping power P_p was about 5 W.

To detect the electromagnetic radiation from the excited system, a microwave antenna of a width $w = 50\text{ }\mu\text{m}$ was attached to the surface of the film as shown in Fig. 1. The microwave signal received by the antenna was fil-

tered within a window of 3.9-4.5 GHz and was amplified by a low-noise microwave amplifier.

The condition $\mu(P_p) = \mu_c = 2\pi\hbar f_{\min}$ corresponds to another threshold value of the pumping power $P_p = P_2$. Above this value an essential part of magnons condensates close to the bottom of the magnon spectrum, f_{\min} . This point of the magnon spectrum, corresponding to the so-called backward volume magnetostatic waves, i.e., $\mathbf{k} \parallel \mathbf{H}_0$, is doubly degenerated (note \mathbf{k}_{\min} and $-\mathbf{k}_{\min}$ in Fig. 2) with $k_{\min} \cong 3 \times 10^4\text{ cm}^{-1}$. We experimentally determine the critical power P_2 by detecting the electromagnetic radiation from the bottom of the magnon spectrum by the antenna. Two spectral lines can be observed in such a case. First one at f_{\min} corresponds to the direct interaction of the magnon condensate with antenna. This line is rather weak, since the antenna is much wider than the wavelength of the corresponding spin waves $w \cdot k_{\min} \gg 1$ and different spatial parts of the condensate having different phases compensate each other. Second line at $2f_{\min}$ is mainly created by the process of two-magnon confluence (\mathbf{k}_{\min} and $-\mathbf{k}_{\min}$) into uniform precession of magnetization ($\mathbf{k} = 0$) as indicated in Fig. 2 by the dash arrows. This radiation can be efficiently detected by the antenna since the phase of the precession with the double frequency is almost uniform over the antenna. This spectral line was used in our experiment to detect the formation of BEC. The sensitivity of the developed detection scheme in the frequency interval 3.9-4.5 GHz was about 10^{-14} W .

Figure 3 shows the BEC radiation power dependence on the applied pumping power. When the pumping power exceeds the critical power P_2 (close to 1 W), the radiation power (at the frequency $2f_{\min}$) increases tremendously from the value below the sensitivity of the detect-

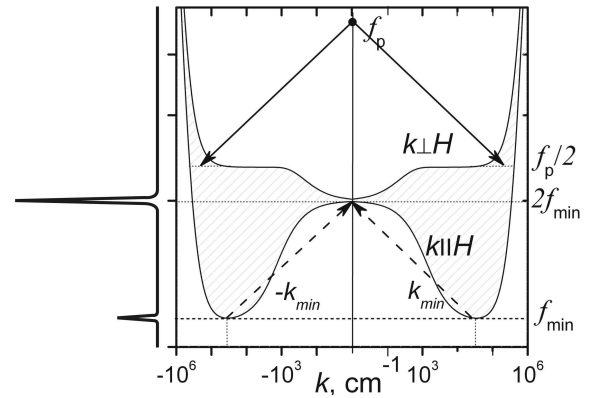


FIG. 2: Processes of parametric pumping used for magnon excitation and of magnon confluence used for the detection of BEC. Right: magnon spectrum for $20\text{ }\mu\text{m}$ thick, magnetized in-plane YIG film ($H_0 = 800\text{ Oe}$). Solid arrows illustrate the parametric pumping process, dashed arrows indicate the confluence of the magnons from the bottom of the spectrum, used for detection of BEC. Left: sketch of the signal detected by the antenna.

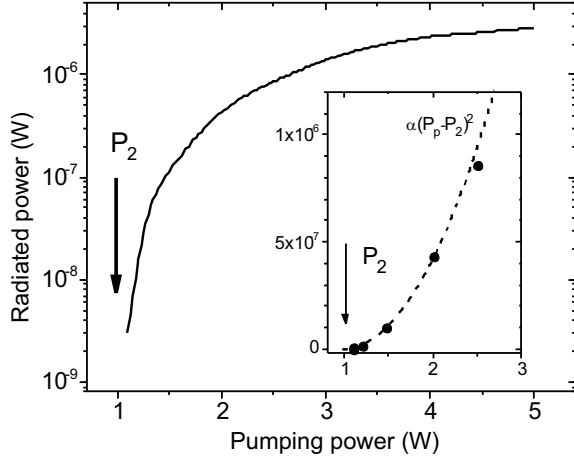


FIG. 3: The power of the detected radiation at $2f_{\min}$ versus the pumping power (note the logarithmic scale). Inset: the onset of the radiation shown in the linear scale, illustrating the procedure for determination of the threshold P_2 .

ing scheme ($< 10^{-14}$ W) to the value of about 10^{-9} W. The critical power P_2 was determined by the extrapolation of the intensity of the detected radiation to zero, as shown in the inset of Fig. 3. The radiation power at double frequency from a magnetic medium is proportional to the second time derivative of the magnetization of the medium squared $\propto (d^2\mathbf{M}/dt^2)^2$. In the case of condensate the derivative is proportional to the density of the condensate magnons $d^2\mathbf{M}/dt^2 \propto N_{\mathbf{k}_{\min}}$ [10]. As the result, the radiation power at $2f_{\min}$ is proportional to $N_{\mathbf{k}_{\min}}^2$, or close to the threshold to $(P_p - P_2)^2$. As seen from Fig. 3, the experimental data nicely corroborate this theoretical prediction. For higher pumping powers the measured function deviates from the quadratic dependence with the tendency to saturate.

Figure 4 demonstrates the dependence of the critical powers P_1 and P_2 on Δf . One can see that the threshold P_1 of the incoherent parametric pumping of magnons increases linearly with Δf in accordance with the theory of incoherent noise pumping [15, 16, 17]. On the contrary, the critical power P_2 of the BEC formation does not depend essentially on Δf , especially for $\Delta f \geq 10$ MHz, where P_2 is nearly constant. This result can be explained using kinetic equation for magnons [10]. The balance between the excited and relaxing magnons at the point of the BEC transition can be represented as

$$\frac{(\gamma h_c)^2}{\Delta f} \frac{k_B T}{8\pi^4 \hbar} \int_{\Delta f} \frac{|V_{\mathbf{k}}|^2 d^3 k}{f_{\mathbf{k}} - f_{\min}} \simeq \frac{1}{\tau_N} [N(\mu_c, T) - N(0, T)], \quad (1)$$

where γ is the gyromagnetic ratio, h_c is the critical amplitude of the pumping field ($P_2 \propto h_c^2$), $V_{\mathbf{k}}$ is the magnon pair coupling strength with the microwave field, τ_N is the relaxation time of magnons, and $N(\mu_c, T)$ and $N(0, T)$

are the volume densities of magnons at the point of the BEC transition and at the true equilibrium with the lattice, correspondingly. The left side of Eq. (1) describes the density of parametric magnons excited in the frequency interval $f_p \pm \frac{1}{2}\Delta f$ per unit time. Since for $\Delta f/f_p \ll 1$ $V_{\mathbf{k}}$ can be considered as a constant, the integral in Eq. (1) is just proportional to Δf . Therefore, the left side is approximately independent of Δf and can be expressed as $\text{const} \cdot h_c^2 \propto P_2$. Since the right side of Eq. (1), which is the relaxation term, does not depend on the external pumping at all, one gets that P_2 is independent of Δf .

A slight increase of P_2 with the lowering of Δf below 10 MHz (see Fig. 4) can indicate the growing role of phase relations between the pump field and the parametric pairs, which is not taken into account in Eq. (1). The mechanism of phase mismatching restricts the nonlinear spin-wave excitation and therefore decreases the number of the excited magnons at a given power of the applied microwave field.

Note that the BEC of magnons occurs in our experiment at rather large supercriticalities $P_2/P_1 \gg 1$. Our current setup does not allow increasing Δf above 17 MHz. However, extrapolating the data in Fig. 4 to larger Δf , we can assume that P_1 and P_2 will be close to each other at $\Delta f \simeq 200$ MHz and the BEC transition can be achieved without strong “overheating” of the parametrically excited magnons. This assumption is in qualitative agreement with the theory of BEC of magnons under incoherent pumping [10]. The theory predicts a possibility of BEC even in the case when the spectrum of the incoherent pumping is so wide that threshold P_1 is never achieved. In this case the magnon system will resemble the famous Frohlich’s model [2] predicting long-range coherence and energy storage to the lowest level in the living systems.

In conclusion, we have experimentally investigated Bose-Einstein condensation of magnons pumped by an

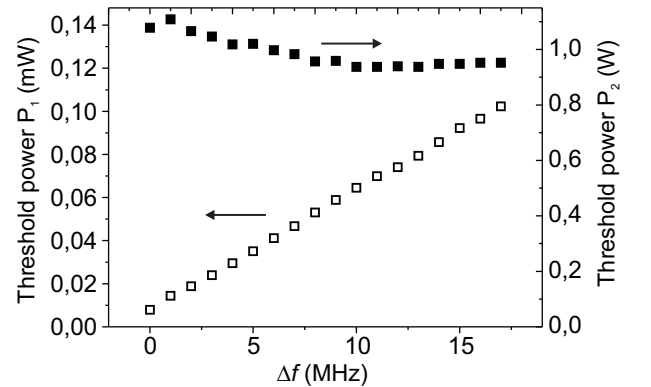


FIG. 4: Dependencies of the two thresholds P_1 (\square) and P_2 (\blacksquare) on the spectral width Δf . Note the linear dependence for P_1 and the almost constant value for P_2 .

incoherent pumping with a variable spectral width Δf . We show experimentally that the BEC formation in a quasi-equilibrium gas can be achieved under such conditions and the power radiated by the condensate scales as $(P_p - P_2)^2$ above the BEC threshold P_2 in agreement with the theory. We demonstrate that while the threshold of incoherent parametric pumping grows proportionally to Δf , P_2 is essentially independent on Δf .

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